Appendix B – Flow Duration Curve/Quantile Regression Analysis of Effects of Urban Hydrology

Introduction

The flow duration curve (FDC) provides a probabilistic description of stream flow at a given location. FDCs can be used in conjunction with flow rating curves for a constituent of interest to compute estimates of average loads. In the analysis described below, quantile regression analysis (QR) is used to estimate two FDCs for both the Northeast Branch (NEB) and Northwest Branch (NWB) tributaries of the Anacostia River, (USGS gage stations 01649500 and 01651000, respectively) representing pre- and post-development hydrology. Second, flow rating curves for sediment for current land sources are computed from 1995 – 2004 monitoring data using output from the USGS ESTIMATOR model (see Appendix A). Finally, the FDC and rating curve functions are used to estimate mean annual loads for both of these tributaries, for pre- and post-watershed hydrology, assuming current watershed land sources for sediment (Schultz, 2006). It is found that approximately 75% of current loads can be attributed to the alterations in watershed hydrology which have taken place over the past 65 years.

Changes in the Flow Duration Curve (FDC) due to urbanization

A FDC can be defined as a function which gives flow, q, as a function of probability, p, i.e. q = q(p), where p is the probability that the specified flow is exceeded. Empirical FDCs are graphs, or alternatively tables, constructed from a set of flow measurements, q_i , made over a given interval of time, ranked from largest to smallest value, with each corresponding p giving the percentage of days for which the flow value was equaled or exceeded. For example, Figure 1 shows two FDCs constructed from daily mean flow data for the NWB for the years, 2002 and 2003. The y-axis of this graph gives daily mean flow observations, q, ordered by rank, in cubic feet per second (cfs), and the x-axis gives the probability of exceedance of each corresponding flow value. For example, the point, (p,q) = (20%, 128) on the graph of 2003 daily flows indicates that during that year, daily mean flows exceeded 128 cfs only 20% of the time.







Daily mean stream flows over a specific time period, and their corresponding FDCs, are determined by both meteorological conditions and by the watershed's hydrologic characteristics. For a given stream, two FDCs constructed for two different time periods will differ because of differences in precipitation and other meteorologic factors. However, changes in watershed conditions can also contribute to changes in the FDC over time. If a watershed becomes highly urbanized, the increase in impervious surface causes increases in storm water runoff and decreases in infiltration and ground water recharge. Thus, the FDC of an urbanized watershed tends to have higher "high flows", representing storm conditions, and lower "low flows", representing base flow conditions, than a similar watershed that has not undergone development.

FDCs are a potentially important tool for quantifying and studying the effects of urbanization on streams, because they respond to changes in a watershed's hydrologic characteristics and because they can be constructed using the type of flow data that is most readily available at most locations, that is, daily means. However, variations in meteorological conditions occur over fairly large time scales, often obscuring long-term time trends caused by a changing watershed hydrology. To detect and quantify changes in the FDC over time, one strategy is to construct FDCs based on data from relatively long time intervals. Figure 2 and Figure 3 show the results of such a computation for the NEB, which has undergone significant urbanization over the period of record for daily flow data, 1939 to present. In these graphs, FDCs are plotted for each complete decade for which data are available. Figure 2 shows the high-flow portion of the decadel FDCs, and Figure 3 shows the low-flow portion of these curves. The decade with the highest high flows is 1970, followed by 1990, 1980, 1950, 1940, and 1960. The decade with the lowest low flows is 1960, followed by 1980, 1990, 1940, 1950, 1970. It is evident from these graphs of decadal FDCs that, as expected, the high-flow portion of the FDC tends to be higher in later decades, and the low-flow portion tends to be lower. However, it is also evident that basing

an analysis on a single pair of decadal FDCs is not particularly reliable, because in this example, some pairs in the series, 1940, 1950, 1960, 1970, 1980, 1990, fail to exhibit the long-term trend.



Figure 2





Figure 3

Calculation of mean sediment loads using FDC and rating curve

The FDC can be used to calculate a mean load for a constituent of interest in cases where a flow rating curve for that constituent is available. A flow rating curve is a formula, or alternatively a table, which gives constituent concentration, C, as a function of flow, i.e., C = C(q). The flow rating curve and the corresponding FDC can be used to compute a mean load for the constituent of interest by integrating the product of the two functions over the interval, $0 \square p \square 1$ (Yorke and Herb, 1976, referencing Miller, 1951), that is,

Load =
$$I q(p) C(q) dp$$
 Equation B-1

For the NEB and NWB tributaries of the Anacostia, flow rating curves for suspended sediment, or "sediment rating curves", can be constructed from data collected during the time period, 1995 – 2004, at the USGS gage stations, 01649500 and 01651000, tabulated in Appendix A. The USGS ESTIMATOR model, which computes multiple least squares regression equations to predict constituent concentrations based on flow, season, and time, can be used to construct flow rating curves by restricting the analyses to flow only. The NEB and NWB flow rating curves for the logarithm of suspended sediment are plotted in Figures 5 and 6 of Appendix A (see plots labeled "ESTIMATOR – Flow variables only"). The sediment rating curves for the NEB and NWB are given by

$$\ln(C_{\text{NEB}}) = 3.61 + 1.18 (\ln(q) - 4.89) + 0.058 (\ln(q) - 4.89)^2$$
 Equation B-2

$$\ln(C_{\text{NWB}}) = 3.58 + 1.21 (\ln(q) - 4.37) + 0.103 (\ln(q) - 4.37)^2$$
 Equation B-3

Coefficients for these equations were not taken directly from the ESTIMATOR-computed regression equation coefficients, which are computed for log C, but rather were obtained via quadratic curve fitting (using Microsoft EXCEL's SOLVER analysis tool) to ESTIMATOR's daily predictions for C. This was done because the ESTIMATOR daily predictions for C include the correction factor necessary to remove the numerical bias introduced by basing the regression analysis on the logarithm of concentration.

Quantifying time trends in the FDC with quantile regression

As discussed above, urbanization is expected to change the tail ends of the FDC, the high flow and the low flow portions of the curve, but not necessarily have an impact on mid-range flows. What's more, it is the changes in high flows and low flows that are of greatest interest to hydrologists, since high flows are responsible for most stream bank and stream channel erosion, and low flows are an indication of ground water recharge and of aquifer levels. However, statistical methods commonly used to determine the response of a dependent variable, such as daily stream flow, to changes in a predictor variable, such as time, are focused on changes in mean values rather than changes in extreme values. Therefore, standard methods such as least squares regression are not particularly suited for detecting and quantifying changes expected to occur in the FDC due to urbanization.

Quantile regression is a type of statistical analysis that has been developed to study changes in all portions of a probability distribution due to changes in a predictor variable. For example, it can be used study changes in the tail ends of the FDC, say, changes in the 95th percentile value flow, over time. The method was originally developed by econometricians (Koenker and Bassett, 1978; see Koenker and Hallock, 2000 for a review) but has recently found use by ecologists and other environmental scientists (Cade and Noon, 2003). In quantile regression, a regression equation for the response variable, Y, as a function of the predictor variables, X, can be calculated for any percentile, 9, of interest. Analogous to standard least square regression, where equation coefficients are computed by minimizing the sum of the squares of the residuals, the coefficients of the quantile regression equation are also computed using an optimization approach, by minimizing a weighted sum of the absolute values of the residuals.

In this application of quantile regression to the FDC, the response variable, Y, is daily mean flow, and the predictor variable, X, is time. In the simple linear case considered here, the conditional quantile for percentile, ϑ , denoted as $Q_Y(\vartheta | X)$, is assumed to be a linear function of X, that is,

$$Q_{Y}(\vartheta|X) = \exists_{0}(\vartheta) + \exists_{1}(\vartheta)X$$
 Equation B-4

where

Y	=	daily mean flow (cfs)
Х	=	time (years)
θ	=	percentile value, $0 \square \vartheta \square 1$.

 $Q_{Y}(\vartheta|X)$ is defined formally as the inverse of the conditional cumulative distribution function, that is,

$$Q_Y(\vartheta|X) = F_Y^{-1}(\vartheta|X)$$
 Equation B-5

where the conditional cumulative distribution function is defined as

$$F_{Y}(y \mid X) = Prob(Y < y \mid X) = \vartheta$$
 Equation B-6

Thus, for a given time, X, the conditional quantile is closely related to the FDC if the variable, p = probability of exceedance is identified with (1 - 9).

In this study, quantile regression was applied to daily mean flow data for the period, 1939 through 2003, for both the Northeast and Northwest Branch tributaries of the Anacostia River, and used to construct a set of regression equations for each tributary which can be used to describe the shape of their FDCs as a function of time. That is, for each tributary, a set of equations of the form of Equation A-4 were constructed for a set of values of ϑ in the range, $0 \Box \vartheta \Box 1$. Computations of quantile regression coefficients and other statistics were done using the statistical free-ware package, "R", available at <u>www.r-project.org</u>, and the quantile regression sub-routine, "quantreg", written by Koenker, also available via the R-project web-site.

Results are given in Tables 1 and 2, which contain values of the quantile regression coefficients, \exists_0 and \exists_1 , for selected values of ϑ (or, alternatively, $p = 1 - \vartheta$). These tables also give the lower and upper ranges of the 90% confidence limits for these coefficients, \exists_0^- and \exists_0^+ , and \exists_1^- and \exists_1^+ , as well as the p-values for \exists_1 , i.e. the probabilities that the slope of the trendline is non-zero. The last six columns of Tables 1 and 2 give the flows predicted by the quantile regression equations, Equation A-1, for the year 1939 (x = 0) and 2004 (x = 65). Flows in columns with the "-" superscript were computed using the lower confidence limits for both \exists_0 and \exists_1 , and flows in columns with the "+" superscript were computed using the upper confidence limits. These flows are plotted in Figures 4 and 5.

Estimates of changes in sediment loads due to changes in hydrology

The NEB and NWB FDC's, given in Table 1 and Table 2, estimated for 1939, representing predevelopment hydrology, and for 2004, representing post-development hydrology, are combined with the corresponding flow rating curves for sediment, representing current sediment loading sources in the watershed, to estimate the fraction of the mean annual sediment load which is attributable to changes in hydrology. Table 3 and Table 4 contain the results of the numerical integration of Equation A-1. The annual loads for the QR-estimated 1939 hydrology are 7,660,000 kg for the NEB and 5,952,000 kg for the NWB. The corresponding annual loads for the QR-estimated 2004 hydrology are 31,987,000 kg and 22,147,000 kg. Thus, according to this analysis, approximately 76% of the current annual mean sediment load for the NEB and 73% for the NWB are do to the alteration in watershed hydrology that have occurred during the past 65 years. Alternatively, this analysis implies that if the hydrology of the watershed could be restored to near-natural conditions, then average annual sediment loads would be reduced by approximately 75%.

 Table 1. Northeast Branch quantile regression equation coefficients and predicted 1939 and 2004 flows (cfs)

p (%)	૭ (%)	∃₀(τ)	∃₀ ⁻ (τ)	∃₀⁺(τ)	p-value for ∃ ₁	∃₁(τ)	∃ ₁ ⁻(τ)	∃ ₁ ⁺(τ)	Predicted Flow 1939	Predicted Flow 1939-	Predicted Flow 1939+	Predicted Flow 2004	Predicted Flow 2004-	Predicted Flow 2004+
0.05	99.95	1840	1407	2410	0.0049	28.46	15.31	40.18	1840	1407	2410	3690	2402	5022
0.1	99.9	1554	1195	2141	0.0271	23.41	9.19	38.57	1554	1195	2141	3076	1792	4648
0.2	99.8	1047	952	1626	0.0129	18.64	6.70	22.39	1047	952	1626	2259	1387	3081
0.4	99.6	958	807	1054	0.0071	8.03	5.62	15.10	958	807	1054	1480	1173	2036
0.5	99.5	838	753	967	0.0012	8.78	4.65	11.12	838	753	967	1408	1055	1690
0.6	99.4	786	657	866	0.0003	7.79	5.81	10.75	786	657	866	1292	1035	1564
0.8	99.2	680	562	795	0.0003	6.73	3.81	11.15	680	562	795	1118	810	1520
1.0	99.0	599	504	691	0.0000	6.28	3.76	9.41	599	504	691	1008	749	1303
1.5	98.5	467	427	508	0.0000	5.63	3.59	6.87	467	427	508	833	660	955
2.0	98.0	406	372	446	0.0000	4.25	2.78	5.20	406	372	446	683	553	784
3.0	97.0	317	284	347	0.0000	3.09	2.10	4.03	317	284	347	518	421	610
4.0	96.0	255	229	284	0.0000	2.65	1.64	3.41	255	229	284	428	336	506
5.0	95.0	227	209	242	0.0000	1.92	1.53	2.39	227	209	242	352	309	397
6.0	94.0	195	179	209	0.0000	1.75	1.34	2.25	195	179	209	309	266	355
7.0	93.0	171	162	182	0.0000	1.66	1.29	1.97	171	162	182	278	246	310
8.0	92.0	155	147	165	0.0000	1.49	1.15	1.77	155	147	165	252	222	280
9.0	91.0	145	139	153	0.0000	1.24	0.94	1.47	145	139	153	225	200	249
10.0	90.0	140	131	144	0.0000	0.91	0.74	1.21	140	131	144	198	180	223
15.0	85.0	107	103	111	0.0000	0.41	0.29	0.54	107	103	111	133	122	146
20.0	80.0	90	87	93	0.0035	0.15	0.06	0.24	90	87	93	100	91	108
30.0	70.0	70	68	71	1.0000	0.00	-0.04	0.05	70	68	71	70	65	74
40.0	60.0	55	53	56	0.3300	-0.02	-0.06	0.02	55	53	56	54	49	58
50.0	50.0	44	43	45	1.0000	0.00	-0.03	0.03	44	43	45	44	41	47
60.0	40.0	36	35	36	1.0000	0.00	-0.02	0.02	36	35	36	36	33	38
70.0	30.0	28	28	28	1.0000	0.00	0.00	0.02	28	28	28	28	28	29
80.0	20.0	22	22	23	1.0000	0.00	-0.03	0.00	22	22	23	22	20	23
90.0	10.0	16	16	17	1.0000	0.00	-0.02	0.00	16	16	17	16	14	17
99.0	1.0	8.2	8.0	8.6	0.0017	-0.03	-0.04	-0.02	8.2	8.0	8.6	6.4	5.3	7.3

Table 2. Northwest Branch quantile regression equation coefficients and predicted 1939 and 2004 flows (cfs)

p (%)	9 (%)	∃ ₀ (τ)	∃ ₀ ⁻ (τ)	∃₀ ⁺ (τ)	p-value for ∃ ₁	∃ ₁ (τ)	∃ ₁ ⁻ (τ)	∃ ₁ ⁺ (τ)	Predicted Flow 1939	Predicted Flow 1939-	Predicted Flow 1939+	Predicted Flow 2004	Predicted Flow 2004-	Predicted Flow 2004+
0.06	99.94	986	893	1458	0.0489	15.25	9.92	29.19	986	893	1458	1977	1538	3355
0.1	99.9	922	807	1092	0.0028	14.48	-0.82	17.19	922	807	1092	1863	753	2209
0.2	99.8	893	679	1079	0.5447	2.42	-2.88	11.90	893	679	1079	1051	492	1852
0.3	99.7	719	624	897	0.0738	3.60	-0.22	6.30	719	624	897	953	610	1307
0.4	99.6	672	527	761	0.1105	2.88	-0.50	7.01	672	527	761	860	494	1217
0.5	99.5	566	463	722	0.0635	3.17	0.38	6.40	566	463	722	772	487	1138
0.6	99.4	502	428	603	0.0188	3.27	1.08	5.61	502	428	603	715	498	968
0.7	99.3	452	417	537	0.0029	3.56	1.53	4.73	452	417	537	683	516	844
0.8	99.2	435	372	480	0.0027	3.12	1.67	4.38	435	372	480	638	481	765
0.9	99.1	425	358	466	0.0153	2.42	1.21	4.25	425	358	466	582	437	742
1.0	99.0	390	328	441	0.0059	2.58	0.91	4.47	390	328	441	557	387	731
1.2	98.8	336	287	404	0.0005	2.67	1.49	4.42	336	287	404	510	384	691
1.4	98.6	305	262	339	0.0000	2.77	1.94	3.96	305	262	339	485	388	596
1.6	98.4	267	237	310	0.0000	3.05	1.79	3.84	267	237	310	465	353	560
1.8	98.2	246	229	278	0.0000	2.96	1.93	3.62	246	229	278	438	354	514
2.0	98.0	238	217	262	0.0000	2.57	1.80	3.38	238	217	262	405	333	482
2.5	97.5	203	178	230	0.0000	2.23	1.55	3.06	203	178	230	348	279	428
3.0	97.0	171	153	198	0.0000	2.27	1.50	2.76	171	153	198	319	251	377
3.5	96.5	152	136	167	0.0000	1.93	1.55	2.51	152	136	167	277	236	330
4.0	96.0	135	125	147	0.0000	1.91	1.53	2.25	135	125	147	259	225	293
4.5	95.5	126	115	137	0.0000	1.65	1.31	2.04	126	115	137	233	200	270
5.0	95.0	117	106	124	0.0000	1.50	1.24	1.84	117	106	124	215	187	244
6.0	94.0	97	87	107	0.0000	1.41	1.18	1.75	97	87	107	189	164	221
7.0	93.0	83	77	89	0.0000	1.38	1.12	1.53	83	77	89	172	149	188
8.0	92.0	74	70	79	0.0000	1.22	1.04	1.37	74	70	79	153	138	168
9.0	91.0	69	65	73	0.0000	1.05	0.91	1.22	69	65	73	137	124	152
10.0	90.0	65	61	68	0.0000	0.94	0.82	1.05	65	61	68	125	115	137
15.0	85.0	48	46	51	0.0000	0.60	0.52	0.67	48	46	51	87	80	94
20.0	80.0	39	37	40	0.0000	0.43	0.38	0.47	39	37	40	67	62	71
25.0	75.0	33	32	34	0.0000	0.35	0.32	0.39	33	32	34	56	52	59
30.0	70.0	28	27	29	0.0000	0.31	0.27	0.34	28	27	29	48	45	51
40.0	60.0	21	20	22	0.0000	0.28	0.25	0.30	21	20	22	39	37	41
50.0	50.0	15.8	15.2	16.4	0.0000	0.24	0.23	0.26	16	15	16	32	30	33
60.0	40.0	11.6	11.3	12.0	0.0000	0.22	0.21	0.23	12	11	12	26	25	27

p (%)	૭ (%)	∃₀(τ)	∃₀ ⁻ (τ)	$\exists_0^{ {}^+}\!(\tau)$	p-value for ∃₁	∃₁(τ)	∃ <u>1</u> (τ)	$\exists_1^+(\tau)$	Predicted Flow 1939	Predicted Flow 1939-	Predicted Flow 1939+	Predicted Flow 2004	Predicted Flow 2004-	Predicted Flow 2004+
70.0	30.0	8.8	8.5	9.2	0.0000	0.18	0.16	0.18	8.8	8.5	9.2	20	19	21
80.0	20.0	6.6	6.3	6.8	0.0000	0.12	0.12	0.14	6.6	6.3	6.8	15	14	16
90.0	10.0	4.5	4.4	4.7	0.0000	0.08	0.08	0.09	4.5	4.4	4.7	10.0	9.5	10.5
99.0	1.0	2.3	2.2	2.3	0.0000	0.02	0.01	0.02	2.3	2.2	2.3	3.2	2.9	3.6

	0	1939 Estimates		2004 Estimates			
p (%)	QR-Predicted Flow (cfs)	ESTIMATOR- Predicted Concentration (mg/L)	Predicted Load for Interval, Δp (1000 kg)	QR-Predicted Flow (cfs)	ESTIMATOR- Predicted Concentration (mg/L)	Predicted Load for Interval, Δp (1000 kg)	
			1,004			5,816	
0.05	1840	1222	824	3690	3530	4,675	
0.1	1554	952	866	3076	2660	5,145	
0.2	1047	539	908	2259	1660	4,252	
0.4	958	475	348	1480	887	1,104	
0.5	838	394	273	1408	825	936	
0.6	786	360	429	1292	728	1,420	
0.8	680	295	311	1118	591	1,045	
1.0	599	248	508	1008	510	1,851	
1.5	467	178	318	833	391	1,163	
2.0	406	148	412	683	297	1,343	
3.0	317	107	242	518	204	765	
4.0	255	82	164	428	158	489	
5.0	227	71	122	352	123	335	
6.0	195	58	88	309	104	256	
7.0	171	50	68	278	91	203	
8.0	155	44	57	252	80	160	
9.0	145	41	51	225	70	123	
10.0	140	39	185	198	60	358	
15.0	107	28	114	133	37	164	
20.0	90	23	147	100	26	167	
30.0	70	18	88	70	18	86	
40.0	55	14	54	54	13	52	
50.0	44	11	35	44	11	35	
60.0	36	9	22	36	9	22	
70.0	28	7	13	28	7	13	
80.0	22	5	8	22	5	8	
90.0	16	4	1	16	4	1	
Total:			7,660			31,987	

 Table 3. Quantile regression/rating curve prediction of NEB annual sediment load

	0	1939 Estimates		2004 Estimates				
p (%)	QR-Predicted Flow (cfs)	ESTIMATOR- Predicted Concentration (mg/L)	Predicted Load for Interval, Δp (1000 kg)	QR-Predicted Flow (cfs)	ESTIMATOR- Predicted Concentration (mg/L)	Predicted Load for Interval, Δp (1000 kg)		
			1,081			7,686		
0.1	922	1312	1,035	1863	4619	4,075		
0.2	893	1242	757	1051	1645	1,356		
0.3	719	861	507	953	1387	1,032		
0.4	672	771	374	860	1164	778		
0.5	566	582	253	772	971	606		
0.6	502	480	189	715	853	513		
0.7	452	406	156	683	791	442		
0.8	435	383	144	638	707	358		
0.9	425	369	126	582	608	299		
1.0	390	323	188	557	567	504		
1.2	336	258	138	510	492	420		
1.4	305	223	104	485	454	373		
1.6	267	183	79	465	426	328		
1.8	246	162	68	438	387	274		
2.0	238	155	137	405	343	515		
2.5	203	123	92	348	271	379		
3.0	171	98	65	319	238	287		
3.5	152	83	49	277	193	221		
4.0	135	71	40	259	175	179		
4.5	126	65	34	233	150	142		
5.0	117	59	51	215	134	221		
6.0	97	47	34	189	112	169		
7.0	83	38	25	172	98	132		
8.0	74	33	20	153	84	101		
9.0	69	31	18	137	73	80		
10.0	65	28	61	125	64	248		
15.0	48	20	35	87	40	120		
20.0	39	16	24	67	29	73		
25.0	33	14	17	56	24	51		
30.0	28	12	22	48	20	71		
40.0	21	9	13	39	16	45		
Anacostia Sediment TMDL, Appendix B		В	11					

Table 4. Quantile regression/rating curve prediction of NWB annual sediment loads

		1020 Estimatos			2004 Entimator	
		1939 Estimates			2004 Estimates	-
p (%)	QR-Predicted Flow (cfs)	ESTIMATOR- Predicted Concentration (mg/L)	Predicted Load for Interval, Δp (1000 kg)	QR-Predicted Flow (cfs)	ESTIMATOR- Predicted Concentration (mg/L)	Predicted Load for Interval, Δp (1000 kg)
50.0	16	7	7	32	13	30
60.0	12	5	4	26	11	20
70.0	9	4	3	20	8	11
80.0	7	3	1	15	6	6
90.0	5	3	1	10	5	2
99.0	2	2	0	3	2	0
Total = Annual Ioad:			5,952			22,147





References

Cade, B.S. and B.R. Noon. 2003. A gentle introduction to quantile regression for ecologists.

Koenker, R. and G. Bassett. 1978. Regression Quantiles. Econometrica, 46, 33-50.

Koenker, R. and K.F. Hallock. 2000. Quantile regression – an introduction. Prepared for Symposium on Econometric Tools, *Journal of Economic Perspectives*, December 28, 2000.

Miller, C.R. 1951. Analysis of flow-duration, sediment-rating curve method of computing sediment yield. US Dept. of Interior, Bur. Of Reclamation, 55 p.

Schultz, C.L. 2006. "Changes in flow duration curve and sediment load due to altered urban hydrology in the Anacostia Northeast Branch and Northwest Branch tributaries", presented at AWRA 2006 Annual Water Resources Conference, Baltimore, Maryland, November 8, 2006.

Yorke, T.H. and W.J. Herb. 1976. Urban-area sediment yield – effects of construction-site conditions and sediment control methods. Proceedings, Third Federal Inter-Agency Sedimentation Conference, Denver, Colorado, March 22-25, 1976. (ER-12)